MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

DEM5038 – ENGINEERING MATHEMATICS 3

(DEE)

29 FEBRUARY 2020
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of six pages including cover page and appendix with four questions only.

2. Attempt **ALL** questions. The distribution of the marks for each question is given.

3. Please write down all your answers in the answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

a) Solve the initial value problem given the homogeneous differential equation as follows

$$y'''-3y''+4y=0;\ y(0)=0,\ y'(0)=1,\ y''(0)=-1$$

[13 marks]

b) Find the general solution for the following non-homogeneous differential equation

$$y''+9y=9t^2$$

[12 marks]

Question 2

a) Suppose that $f(t)$ is a periodic function with one period as follows:

$$f(t)=\begin{cases}-t, & -1<t<0\\ t, & 0<t<1\end{cases}$$

i) What is the fundamental period T and angular frequency ω?

ii) State whether $f(t)$ is an even or odd function.

iii) Subsequently, based on part ii), find the Fourier series of $f(t)$.

[Hint: (1) $\int t\cos(at)dt=\frac{\cos(at)}{a^2}+\frac{t\sin(at)}{a}$, (2) $\int t\sin(at)dt=\frac{\sin(at)}{a^2}-\frac{t\cos(at)}{a}$]

[2+1+10 marks]

b) Suppose that $f(t)$ is a non-periodic function given by:

$$f(t)=1,\text{ for }\ 0<t<1$$

i) Rewrite the function $f(t)$ as an odd periodic extension.

ii) Draw the waveform of the function in part i).

iii) Subsequently, find the half-range Fourier Sine series of the function in part i).

[3+3+6 marks]

Continued...

Question 3

a) Find the Laplace transform of the following functions by using tables of Laplace transform pairs and properties:

i) $f(t) = 5e^{-t}\cos(t)$

ii) $f(t) = t\sinh(t)$

iii) $f(t) = 2u(t) - u(t-1) + (t-2)u(t-2)$

iv) $f(t) = \int_0^t \tau e^{\tau} d\tau$

[3+3+3+3 marks]

b) Solve the following differential equation using Laplace transform methods.

$$y''+3y'+2y = e^{-t},\ y(0) = 0,\ y'(0) = 0$$

[13 marks]

Continued...

Question 4

a) A customer survey has been conducted on an electronic product. The customer can either rate the product as good or poor in quality. 15 customers have given a 20% rating of poor to the product. Using the table of binomial distribution function, find the probability that

i. at least 4 of the customers have given a poor rating.

ii. between 5 and 8 (inclusive) customers have given a poor rating.

iii. more than 8 customers have given a poor rating.

[3+3+3 marks]

b) The population in Melaka with monthly household income is normally distributed with mean RM5600 and standard deviation of RM1500. Assume that the household income is directly contributed by monthly wage only. Using the table of normal distribution function,

i. what is the likelihood of the population in Melaka having a minimum wage of RM1200?

ii. what is the likelihood of the population in Melaka having a wage between RM2500 and RM3500?

[4+5 marks]

c) The mean examination score of a certain course throughout the years is 67 marks and normally distributed. In a recent examination of 50 students, a mean score of 75 with standard deviation of 15 marks have been obtained. Perform a statistical test at 5% significance level whether the score is out of norm.

[7 marks]

End of Paper

Appendix

Fourier Series Formulas

Fourier Series

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t)$$

where

$$\omega = \frac{2\pi}{T}$$

$$A_0 = \frac{1}{T}\int_T f(t)dt$$

$$A_n = \frac{2}{T}\int_T f(t)\cos(n\omega t)dt$$

$$B_n = \frac{2}{T}\int_T f(t)\sin(n\omega t)dt$$

Half-Range Fourier Series

$$F(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}t\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}t\right)$$

where

$$A_0 = \begin{cases} 0, & \text{for Sine Series} \\ \frac{1}{L}\int_0^L f(t)dt, & \text{for Cosine Series} \end{cases}$$

$$A_n = \begin{cases} 0, & \text{for Sine Series} \\ \frac{2}{L}\int_0^L f(t)\cos\left(\frac{n\pi}{L}t\right)dt, & \text{for Cosine Series} \end{cases}$$

$$B_n = \begin{cases} \frac{2}{L}\int_0^L f(t)\sin\left(\frac{n\pi}{L}t\right)dt, & \text{for Sine Series} \\ 0, & \text{for Cosine Series} \end{cases}$$

Laplace Transform Pairs of Some Functions

$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
$\delta(t)$	1
$\delta(t-a)$, $a \geq 0$	e^{-as}
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$, $n = 1,2,3,\ldots$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$, $n = 1,2,3,\ldots$

$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sinh(kt)$	$\frac{k}{s^2-k^2}$
$\cosh(kt)$	$\frac{s}{s^2-k^2}$
$u(t-a)$, $a \geq 0$	$\frac{e^{-as}}{s}$
$(t-a)u(t-a)$, $a \geq 0$	$\frac{e^{-as}}{s^2}$

Laplace Transform Properties

Operations	$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
Linearity	$af_1(t)+bf_2(t)$	$aF_1(s)+bF_2(s)$
Time shifting	$f(t-t_0)u(t-t_0), t_0 \geq 0$	$F(s)e^{-st_0}$
Frequency shifting	$e^{at}f(t)$	$F(s-a)$
Time differentiation	$f'(t)$	$sF(s)-f(0)$
	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
Time integration	$\int_0^t f(\tau)d\tau$	$\frac{1}{s}F(s)$
Frequency differentiation	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(\tilde{s})d\tilde{s}$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Normal Distribution

$$Z = \frac{X-\mu}{\sigma}$$

Hypothesis Testing

$$Z = \frac{\overline{X}-\mu}{s/\sqrt{n}}$$